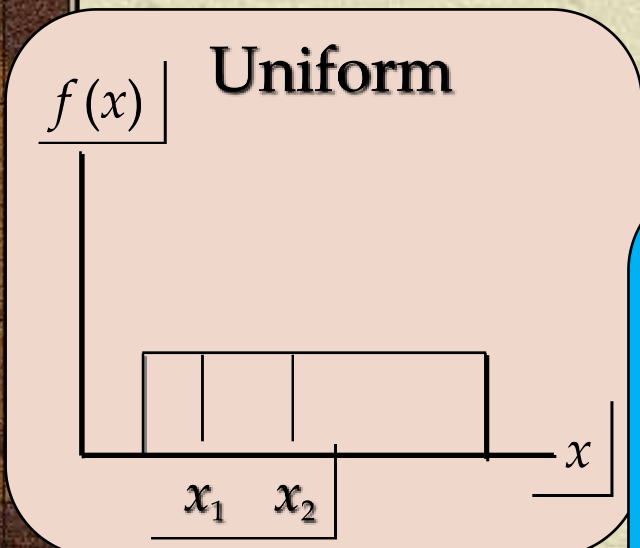
Continuous Probability Distributions

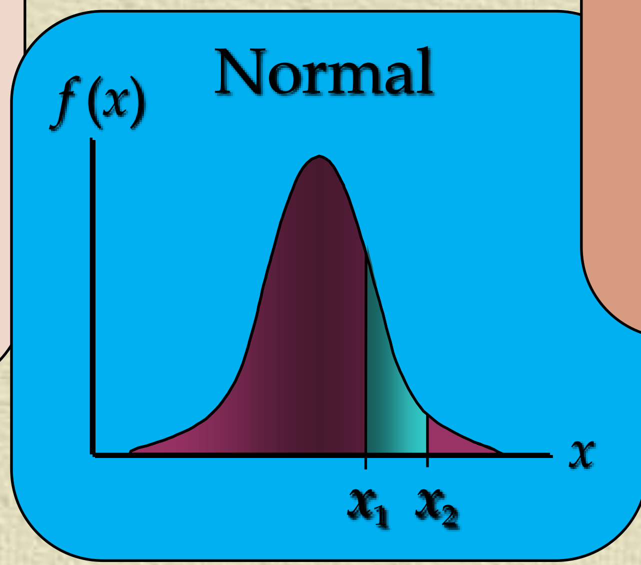
Continuous Probability Distributions

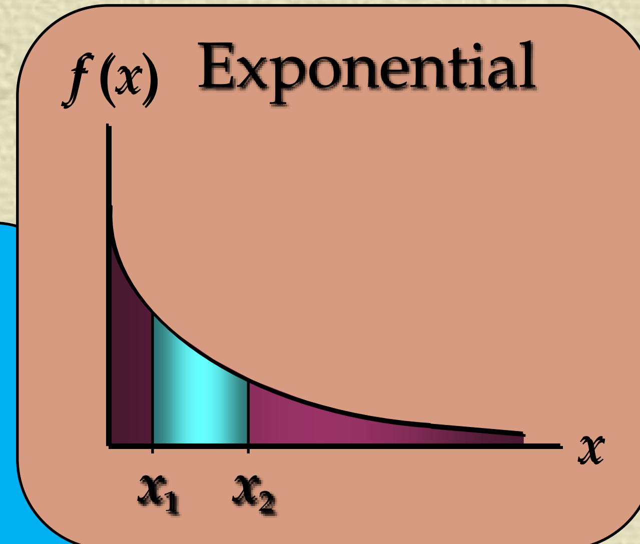
- A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.
- It is not possible to talk about the probability of the random variable assuming a particular value. (the area at a particular point is zero)
- Instead, we talk about the probability of the random variable assuming a value within a given interval.

Continuous Probability Distributions

- The probability of the random variable assuming a value within some given interval from x_1 to x_2 is defined to be the area under the graph of the probability density function between x_1 and x_2.







Uniform Probability Distribution

- A random variable is <u>uniformly distributed</u> whenever the probability is proportional to the interval's length.
- The <u>uniform probability density function</u> is:

$$f(x) = 1/(b - a) \quad \text{for } a \leq x \leq b$$
$$= 0 \quad \text{elsewhere}$$

where: a = smallest value the variable can assume

b = largest value the variable can assume

Uniform Probability Distribution

- Expected Value of x

$$E(x) = (a + b)/2$$

- Variance of x

$$Var(x) = (b - a)^2/12$$

Uniform Probability Distribution

- Example: Slater's Buffet

Slater customers are charged for the amount of salad they take. Sampling suggests that the amount of salad taken is uniformly distributed between 5 ounces and 15 ounces.

Uniform Probability Distribution

- Uniform Probability Density Function

$$f(x) = 1/10 \quad \text{for } 5 \leq x \leq 15$$
$$= 0 \quad \text{elsewhere}$$

where:

x = salad plate filling weight

Uniform Probability Distribution

- Expected Value of x

$$E(x) = (a + b)/2$$
$$= (5 + 15)/2$$
$$= 10$$

- Variance of x

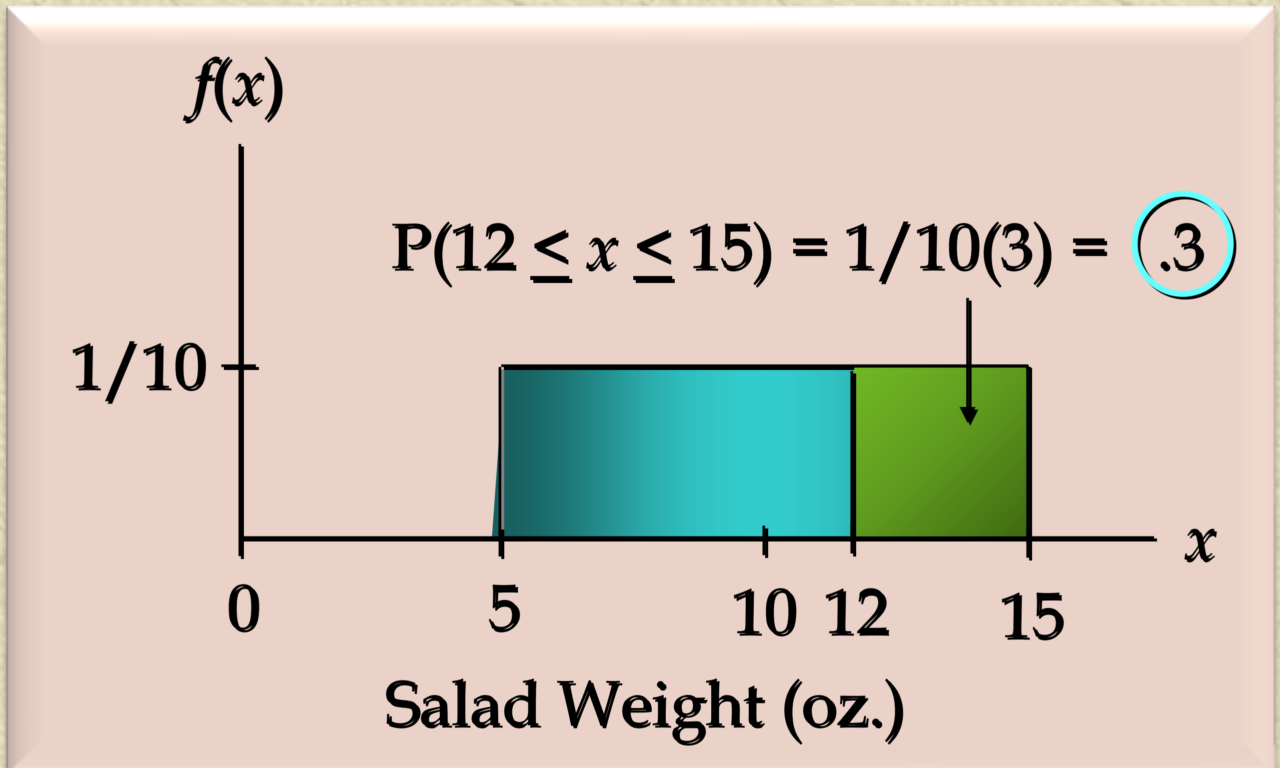
$$Var(x) = (b - a)^2/12$$
$$= (15 – 5)^2/12$$
$$= 8.33$$

Uniform Probability Distribution

- Uniform Probability Distribution for Salad Plate Filling Weight

Uniform Probability Distribution

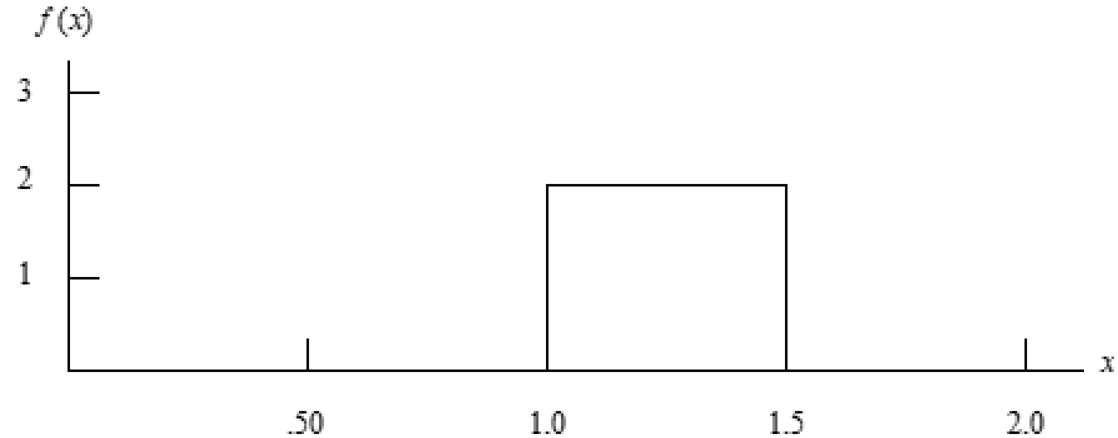
What is the probability that a customer will take between 12 and 15 ounces of salad?



Practice Example

1. The random variable x is known to be uniformly distributed between 1.0 and 1.5.
   a. Show the graph of the probability density function.
   b. Compute $P(x = 1.25)$.
   c. Compute $P(1.0 \leq x \leq 1.25)$.
   d. Compute $P(1.20 < x < 1.5)$.
2. The random variable x is known to be uniformly distributed between 10 and 20.
   a. Show the graph of the probability density function.
   b. Compute $P(x < 15)$.
   c. Compute $P(12 \leq x \leq 18)$.
   d. Compute $E(x)$.
   e. Compute $Var(x)$.

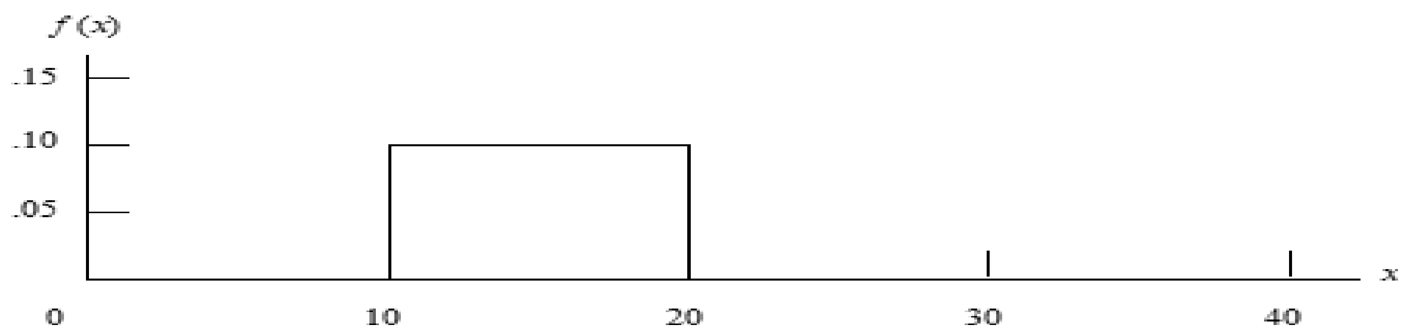
1. a.



b. $P(x = 1.25) = 0$. The probability of any single point is zero since the area under the curve above any single point is zero.

c. $P(1.0 \leq x \leq 1.25) = 2(.25) = .50$

d. $P(1.20 < x < 1.5) = 2(.30) = .60$



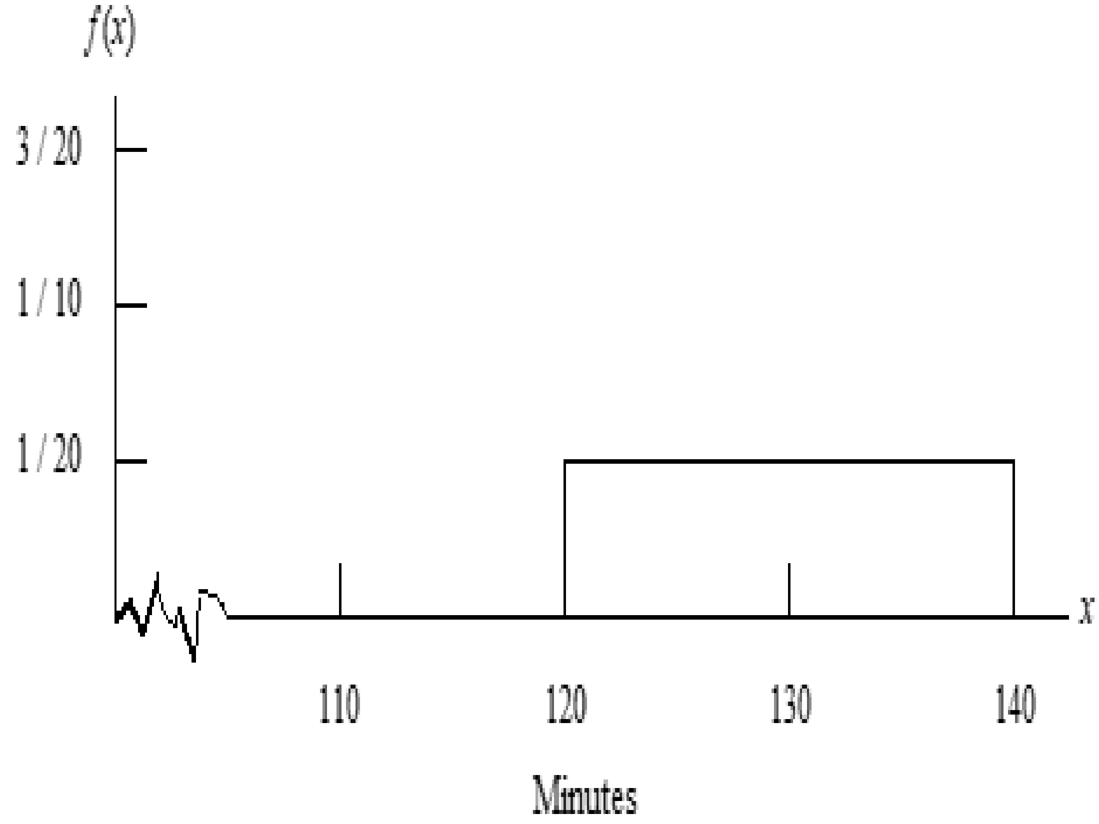
b. $P(x < 15) = .10(5) = .50$

c. $P(12 \leq x \leq 18) = .10(6) = .60$

d. $E(x) = \frac{10+20}{2} = 15$

e. $Var(x) = \frac{(20-10)^2}{12} = 8.33$

3. Delta Airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Tampa. Suppose we believe that actual flight times are uniformly distributed between 2 hours and 2 hours, 20 minutes.
   a. Show the graph of the probability density function for flight time.
   b. What is the probability that the flight will be no more than 5 minutes late?
   c. What is the probability that the flight will be more than 10 minutes late?
   d. What is the expected flight time?



b. $P(x \leq 130) = (1/20)(130 - 120) = 0.50$

c. $P(x > 135) = (1/20)(140 - 135) = 0.25$

d. $E(x) = \dfrac{120 + 140}{2} = 130$ minutes

Area as a Measure of Probability

- The area under the graph of $f(x)$ and probability are identical.

- This is valid for all continuous random variables.

- The probability that x takes on a value between some lower value x_1 and some higher value x_2 can be found by computing the area under the graph of $f(x)$ over the interval from x_1 to x_2.

Normal Probability Distribution

- The normal probability distribution is the most important distribution for describing a continuous random variable.

  - It is widely used in statistical inference.

  - It has been used in a wide variety of applications including:

    - Heights of people
    - Rainfall amounts
    - Test scores
    - Scientific measurements

Normal Probability Distribution

- Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where:

μ = mean

σ = standard deviation

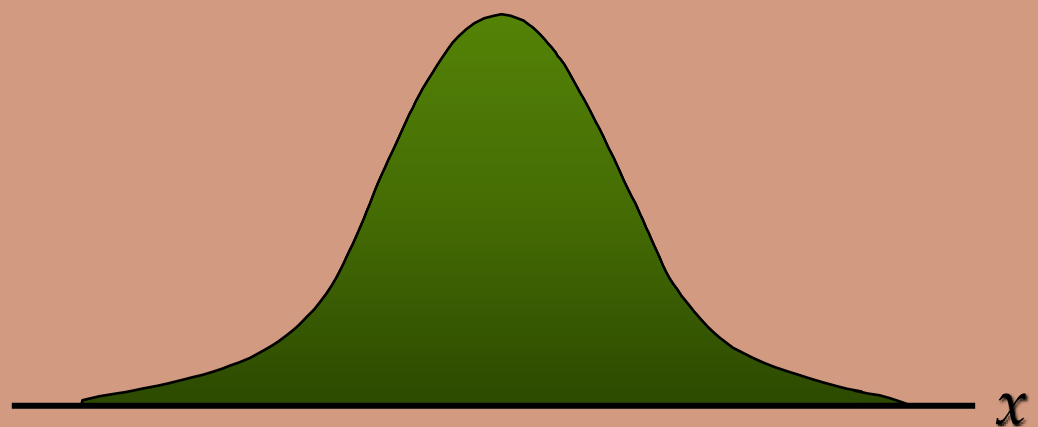
π = 3.14159

e = 2.71828

Normal Probability Distribution

- Characteristics

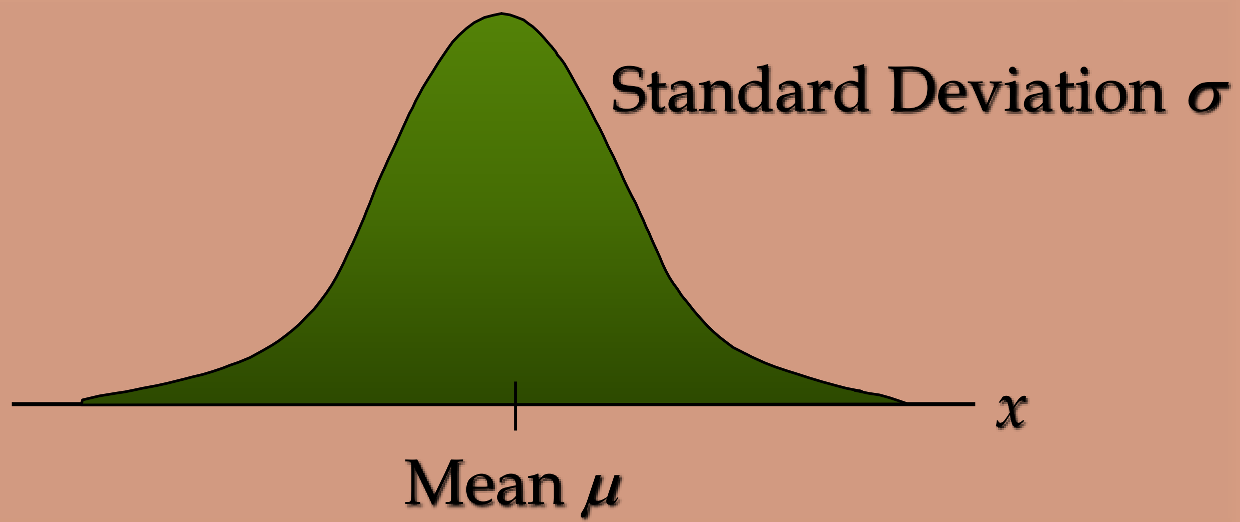
The distribution is symmetric; its skewness measure is zero.



Normal Probability Distribution

- Characteristics

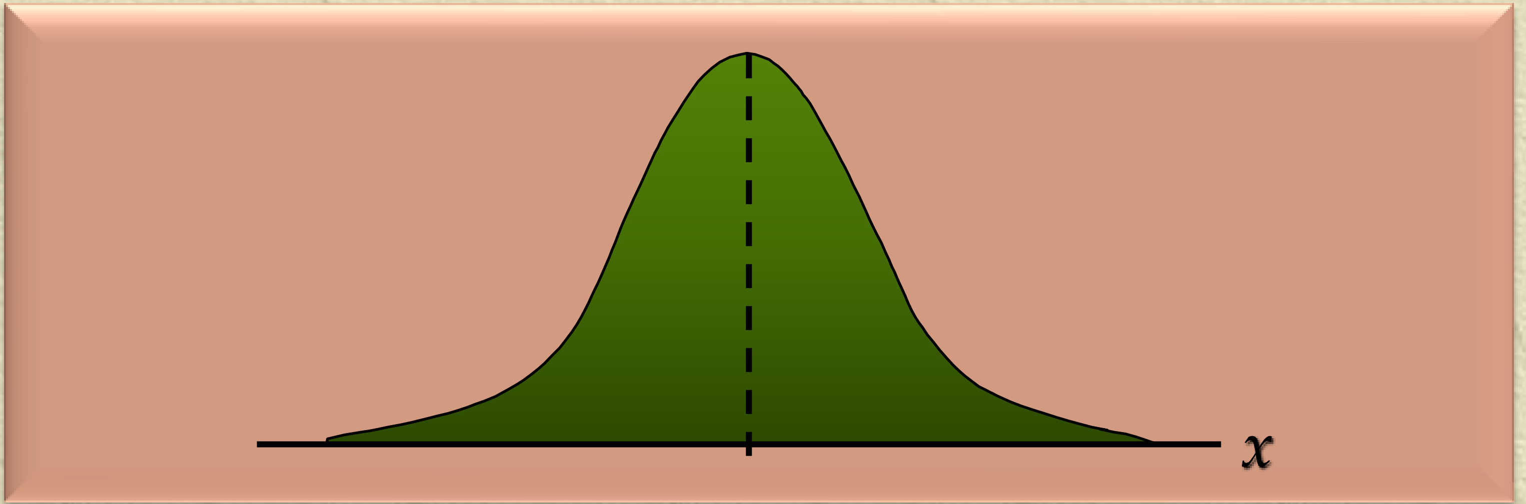
The entire family of normal probability distributions is defined by its <u>mean</u> μ and its <u>standard deviation</u> σ.



Normal Probability Distribution

- Characteristics

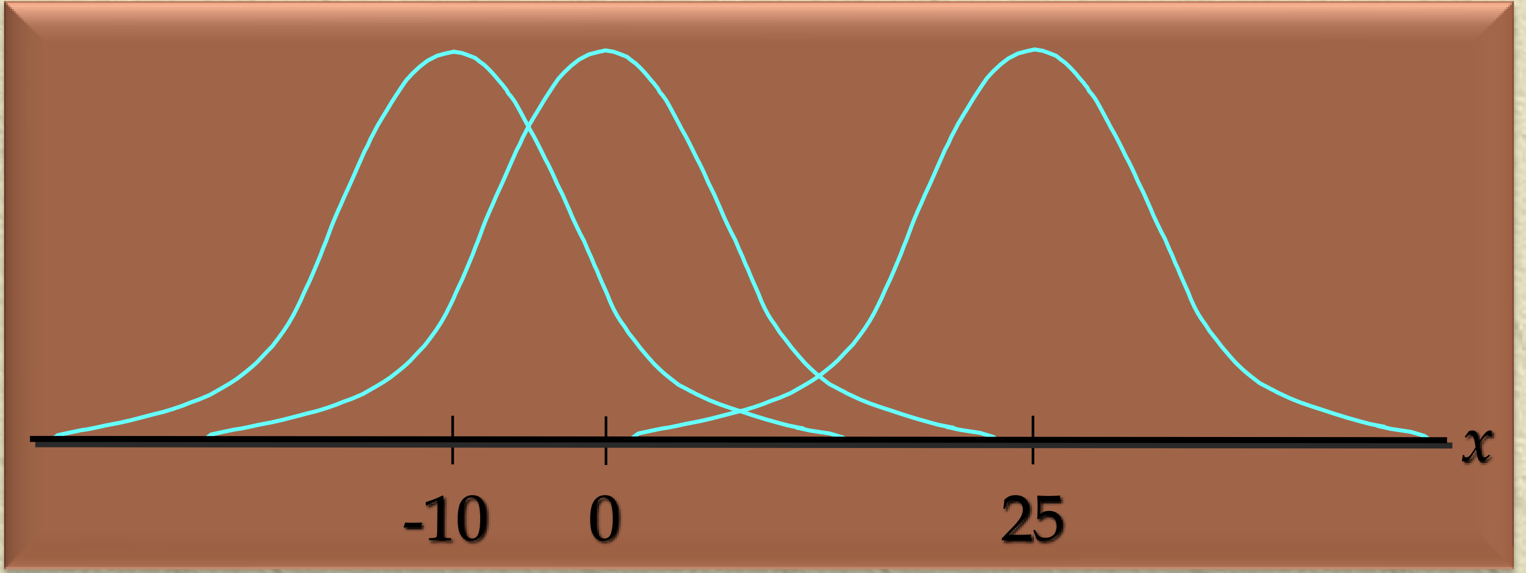
The highest point on the normal curve is at the mean, which is also the median and mode.



Normal Probability Distribution

- Characteristics

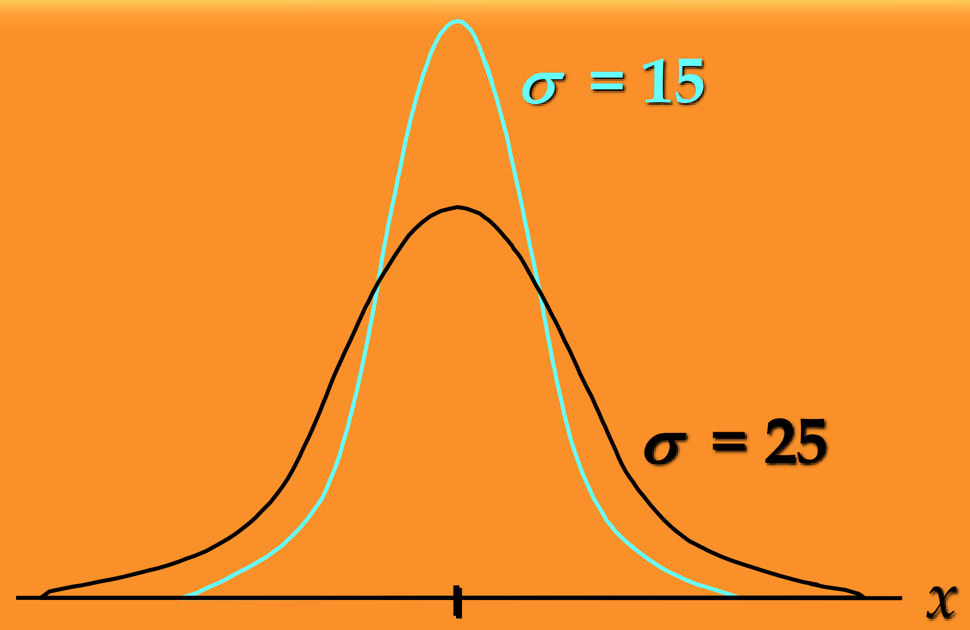
The mean can be any numerical value: negative, zero, or positive.



Normal Probability Distribution

- Characteristics

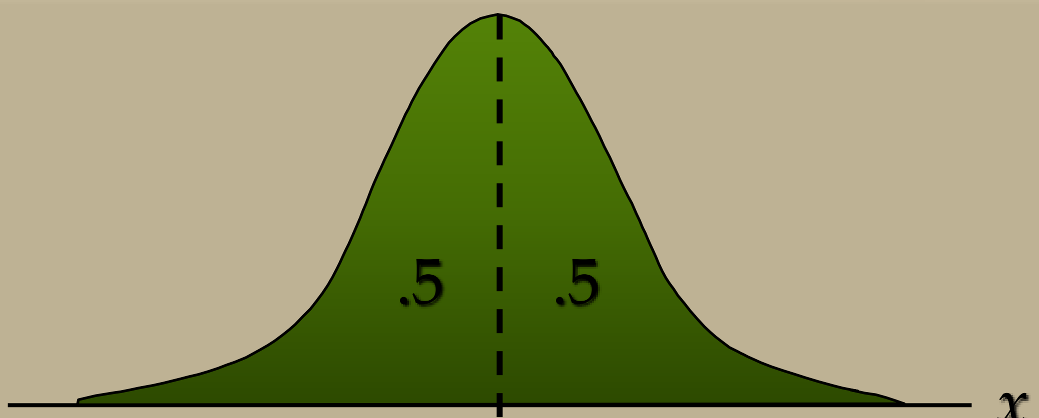
The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



Normal Probability Distribution

- Characteristics

Probabilities for the normal random variable are given by <u>areas under the curve</u>. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



Normal Probability Distribution

- Characteristics (basis for the empirical rule)

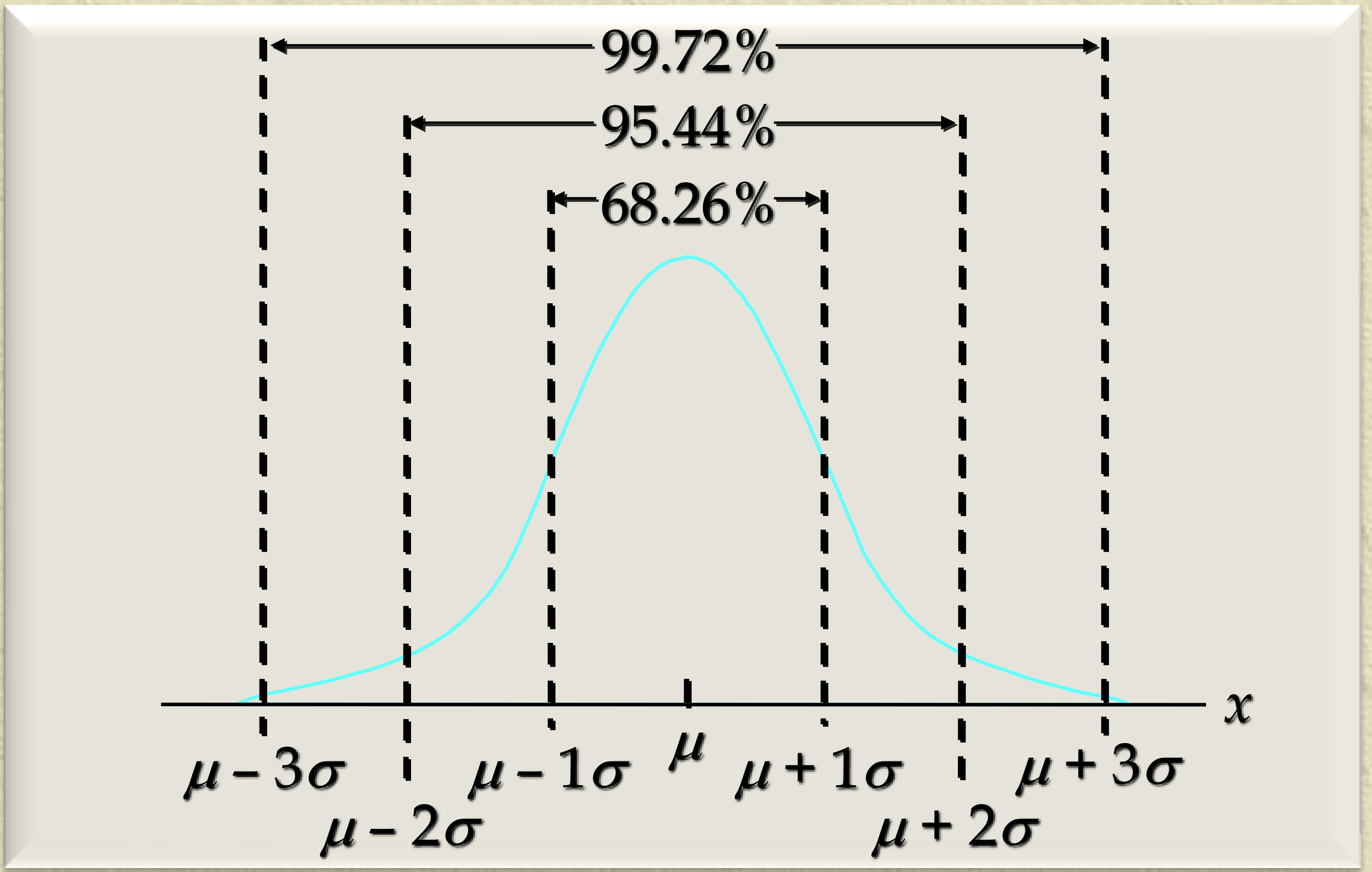
68.26% of values of a normal random variable are within +/- 1 standard deviation of its mean.

95.44% of values of a normal random variable are within +/- 2 standard deviations of its mean.

99.72% of values of a normal random variable are within +/- 3 standard deviations of its mean.

Normal Probability Distribution

- Characteristics (basis for the empirical rule)



Standard Normal Probability Distribution

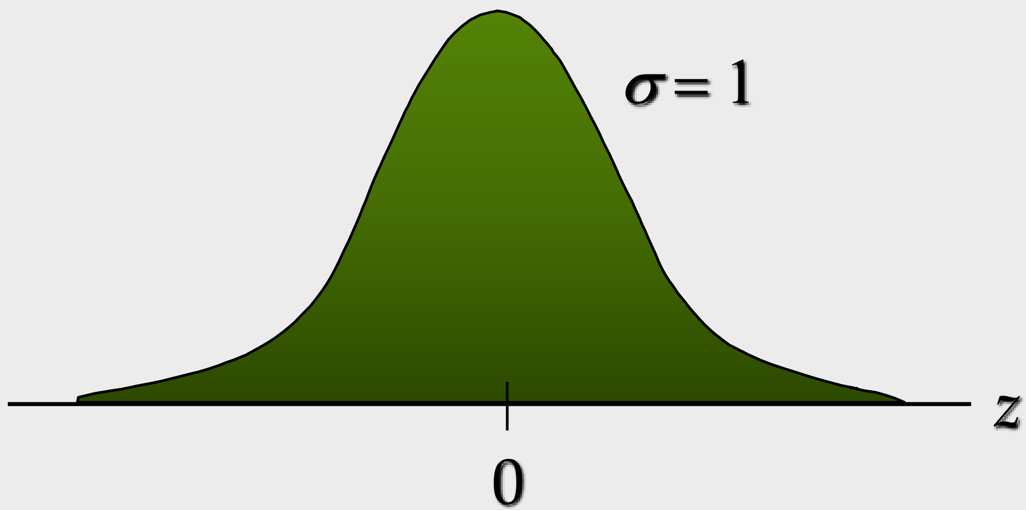
- Characteristics

A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

Standard Normal Probability Distribution

- Characteristics

The letter z is used to designate the standard normal random variable.



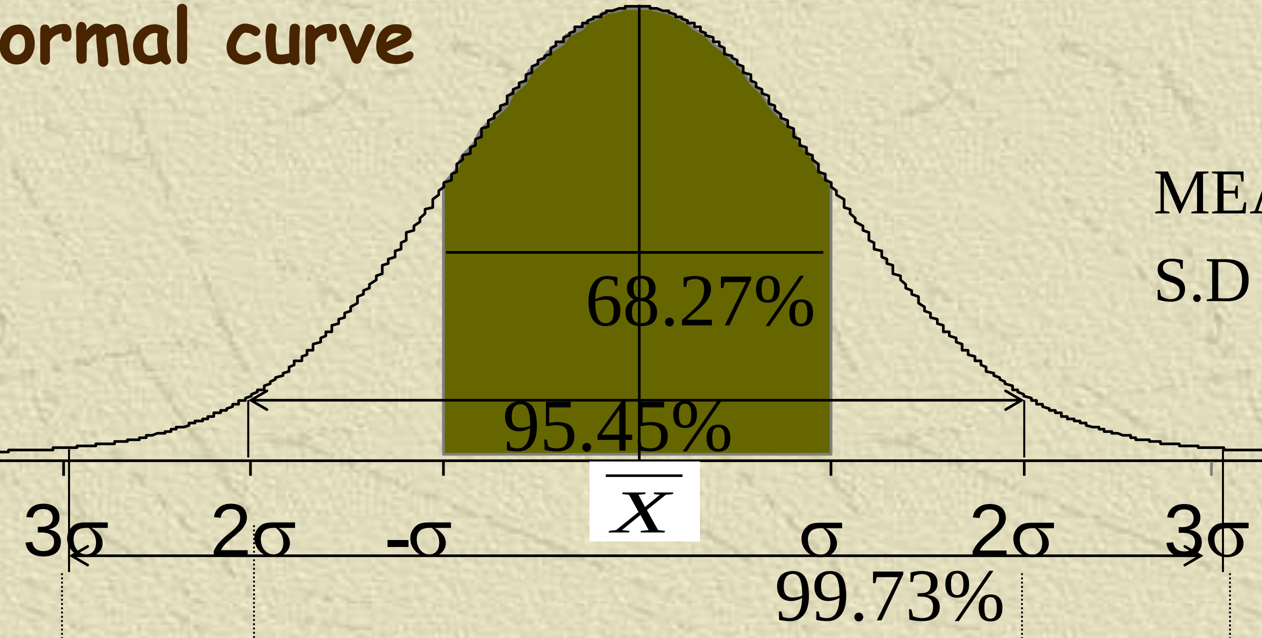
Standard Normal Probability Distribution

- Converting to the Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

We can think of z as a measure of the number of standard deviations x is from μ.

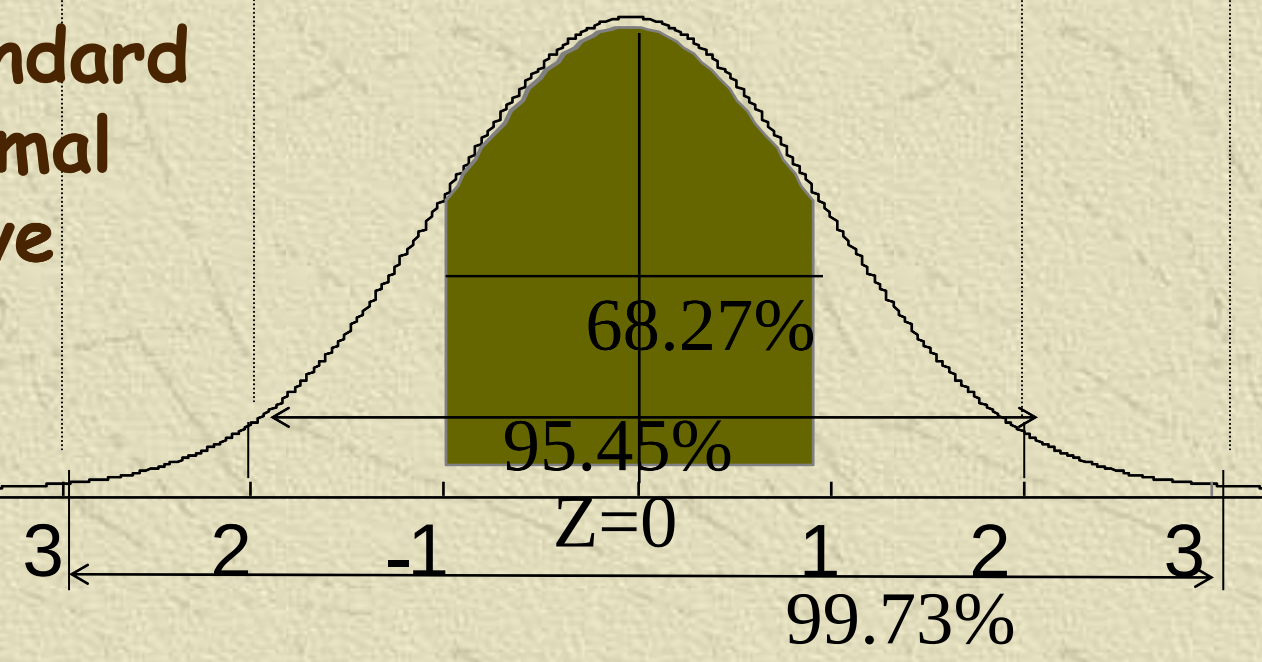
Normal curve



MEAN = $\bar{X}$

S.D = σ

Standard Normal Curve

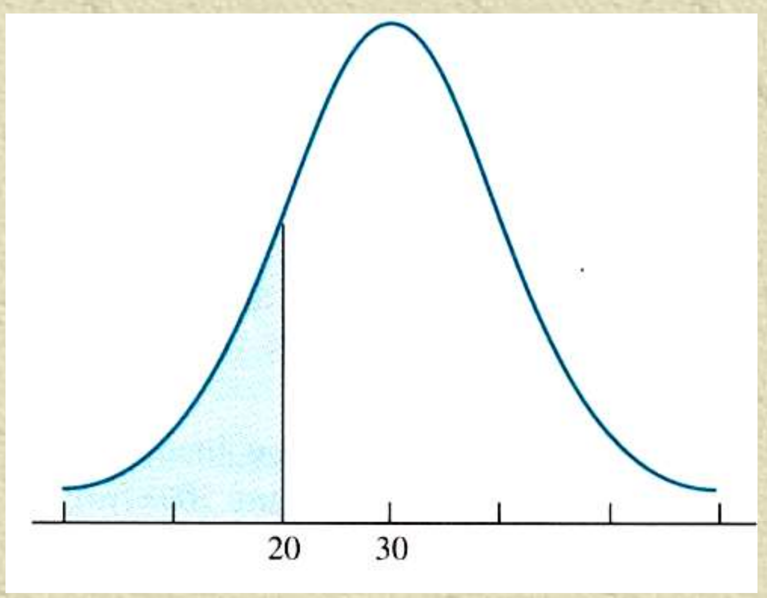


MEAN=0

S.D=1

Example: Finding a Proportion Below the Mean

Assume that the time it takes a bank teller to serve a customer is normally distributed with a mean of 30 seconds and a standard deviation of 10 seconds. What percent of customers are served in less than 20 seconds?



$$Z = \frac{X - \mu}{\sigma}$$

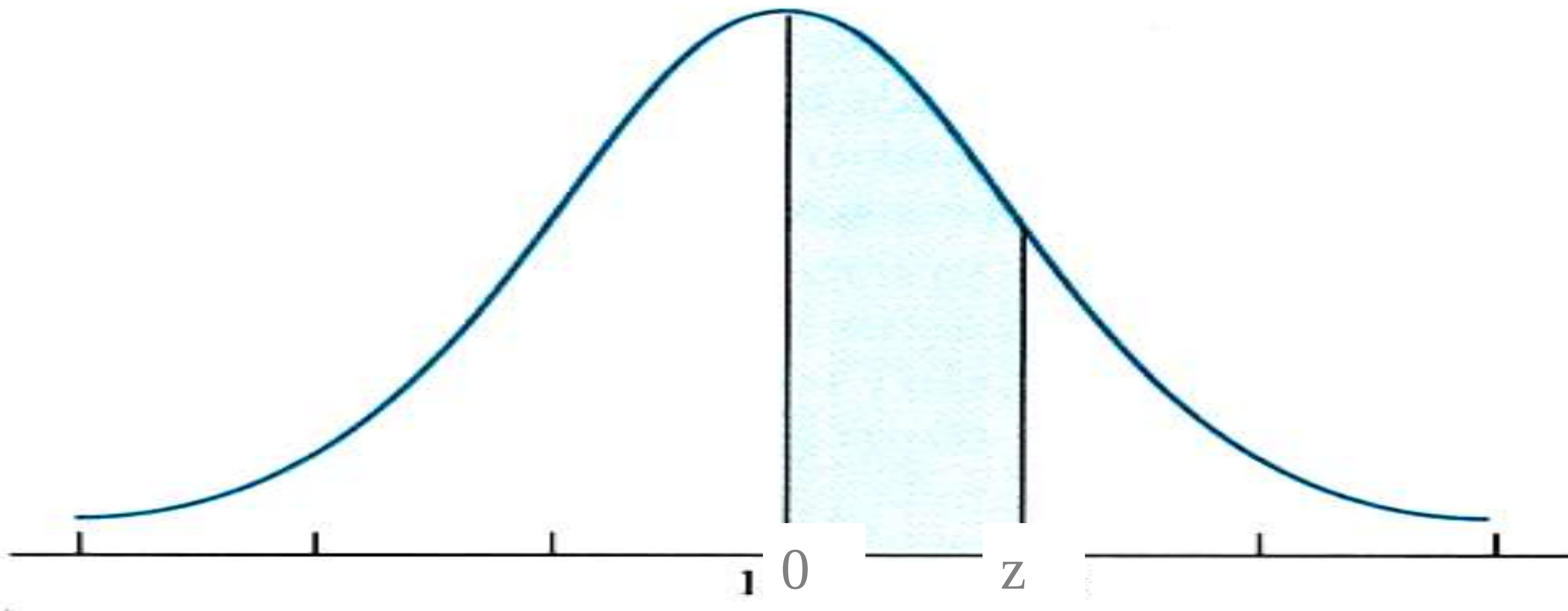
$$X = 20, \mu = 30, \sigma = 10$$

$$Z = \frac{20 - 30}{10} = -1$$

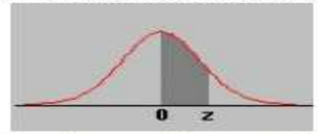
TABLE OF Z- TRANSFORM

- Gives area under curve for +ve side only. [Between 0 to z].
- For area on –ve side, find area on +ve side for the same value (symmetric curve)

$$P(0 \le z \le 1.20) = P(-1.20 \le z \le 0)$$



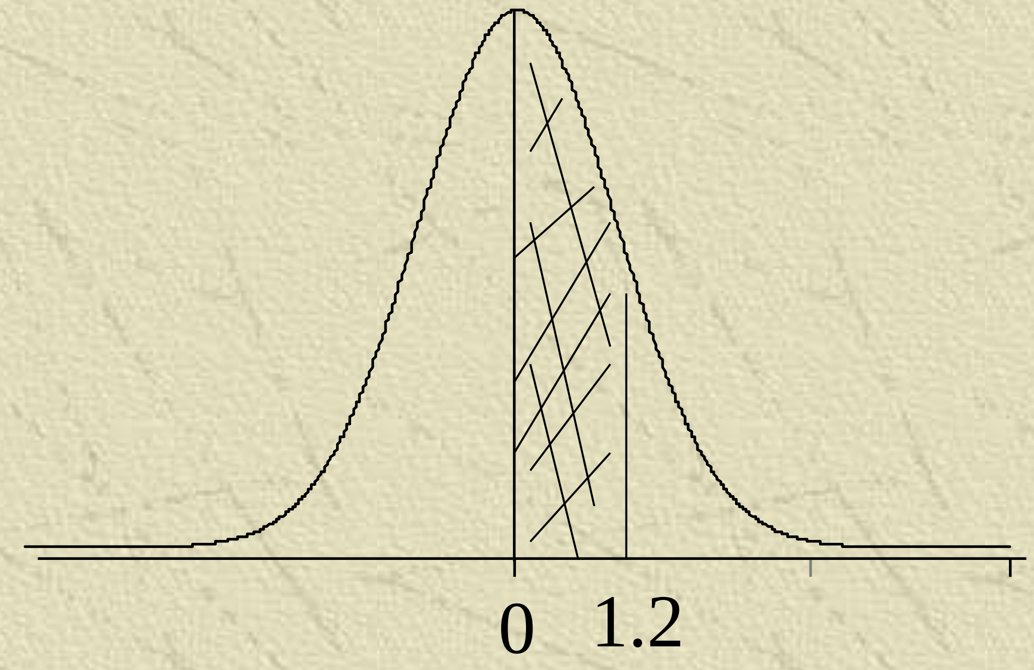
Standard Normal (Z) Table
Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Find area under the normal curve in each of the following cases-

1) P(0 ≤ Z ≤ 1.2)

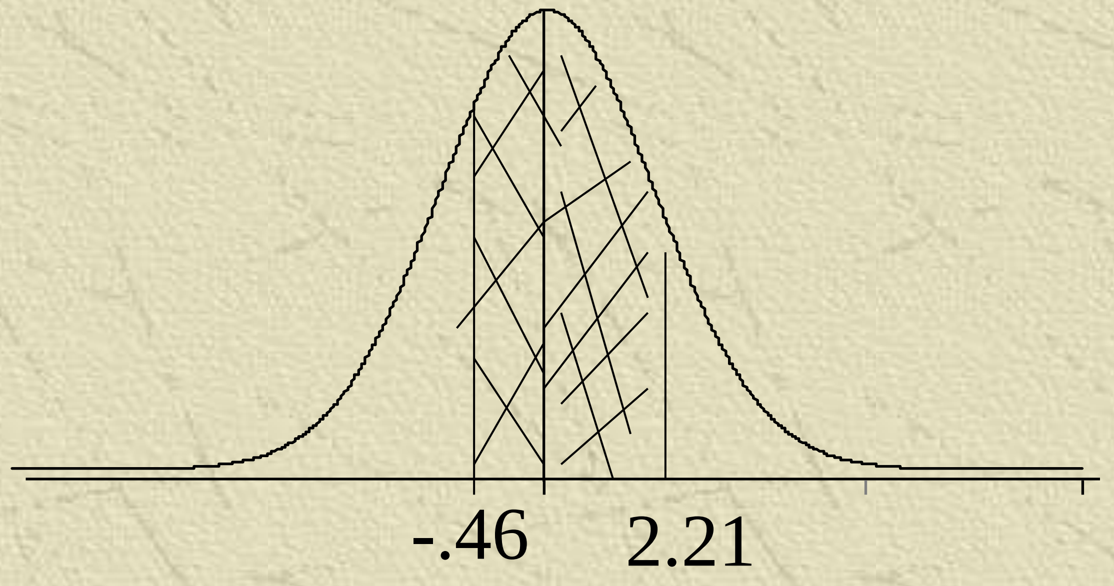


Ans: .3849

2) $P(-.68 \le Z \le 0)$

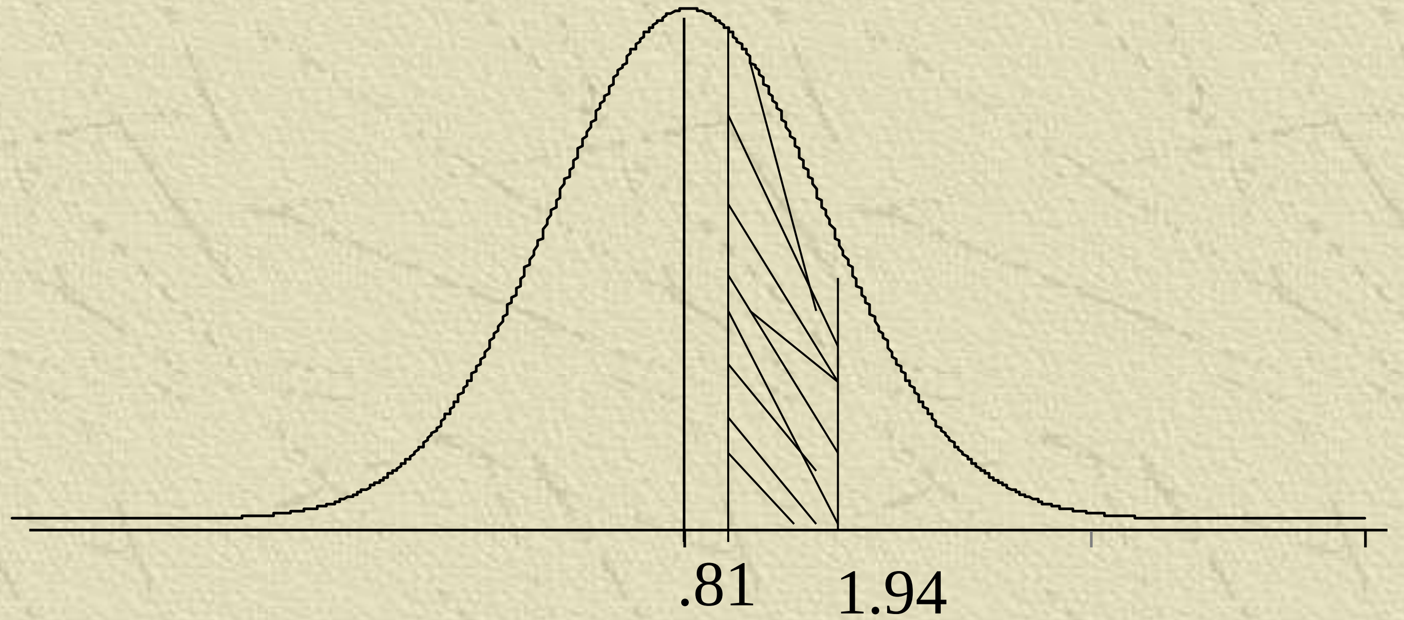
$= P(0 \le Z \le .68) = .2517$

3) $P(-.46 \le Z \le 2.21)$



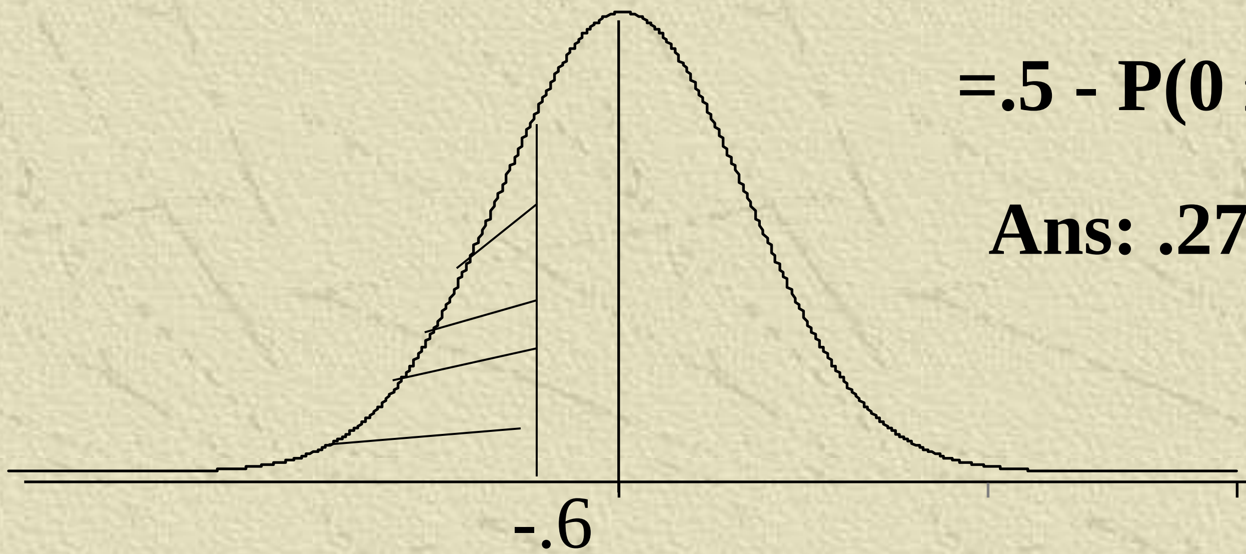
$P(-.46 \le Z \le 0) + P(0 \le Z \le 2.21)$
$= .1772 + .4864 = .6636$

4) P(.81 ≤ Z ≤ 1.94)



P(0 ≤ Z ≤ 1.94) – P(0 ≤ Z ≤ .81)
=.4738 - .2910 = .1828

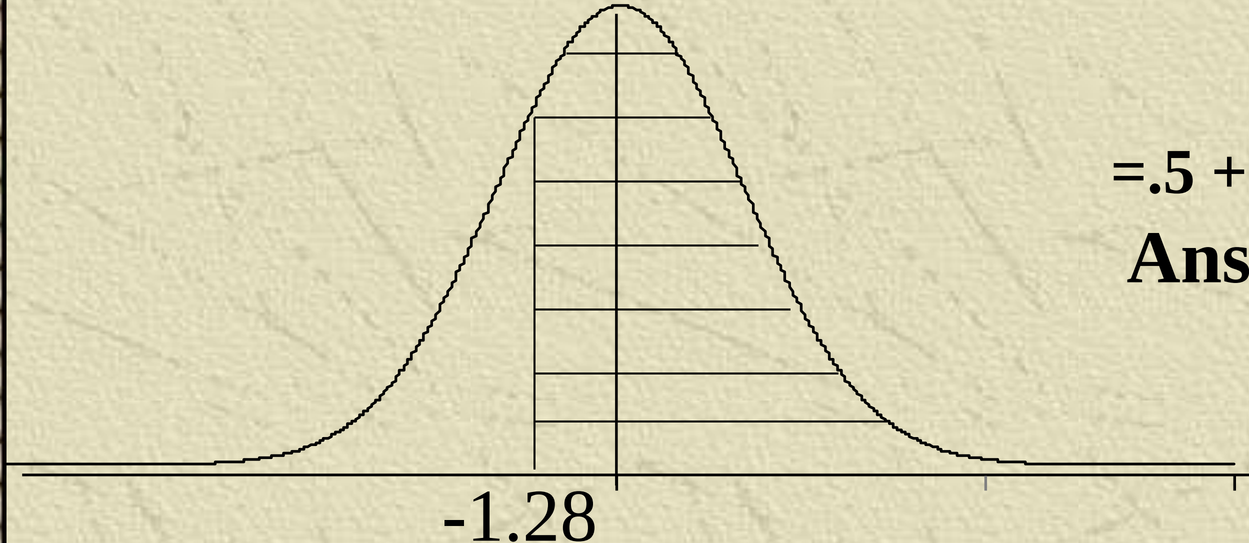
5) TO THE LEFT OF Z = - .6



$= .5 - P(0 \le Z \le .6)$

Ans: .2743

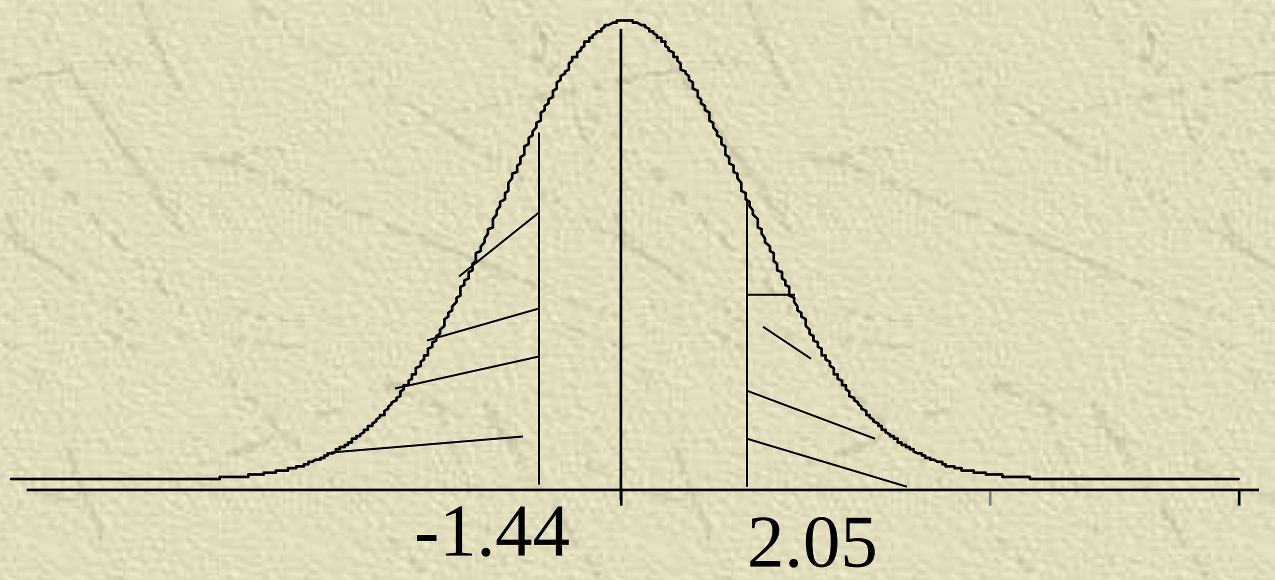
6) TO THE RIGHT OF Z= -1.28



$= .5 + P(0 \le Z \le 1.28)$

Ans: .8997

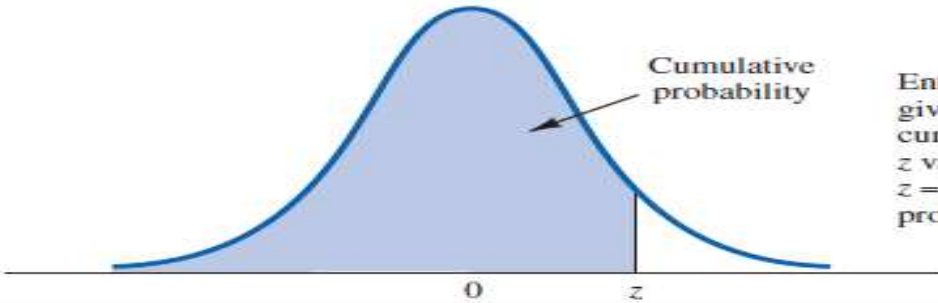
7) RIGHT OF Z= 2.05 AND LEFT OF Z= -1.44



$= 1 - P(-1.44 \leq Z \leq 2.05)$

$= 1 - [P(0 \leq Z \leq 1.44) + P(0 \leq Z \leq 2.05)] = .0951$

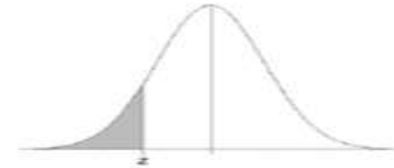
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Probability Distribution

- Example: Pep Zone

Pep Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed.

The store manager is concerned that sales are being lost due to stockouts while waiting for a replenishment order.

Standard Normal Probability Distribution

- Example: Pep Zone

  It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

  The manager would like to know the probability of a stockout during replenishment lead-time. In other words, what is the probability that demand during lead-time will exceed 20 gallons?

$$P(x > 20) = ?$$

Standard Normal Probability Distribution

- Solving for the Stockout Probability

Step 1: Convert x to the standard normal distribution.

$$z = (x - \mu)/\sigma$$
$$= (20 - 15)/6$$
$$= .83$$

Step 2: Find the area under the standard normal curve to the left of $z = .83$.

Standard Normal Probability Distribution

- Cumulative Probability Table for the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
.	.	.	.	.	.	.	.	.	.	.

$P(z \leq .83)$

Standard Normal Probability Distribution

- Solving for the Stockout Probability

Step 3: Compute the area under the standard normal curve to the right of z = .83.

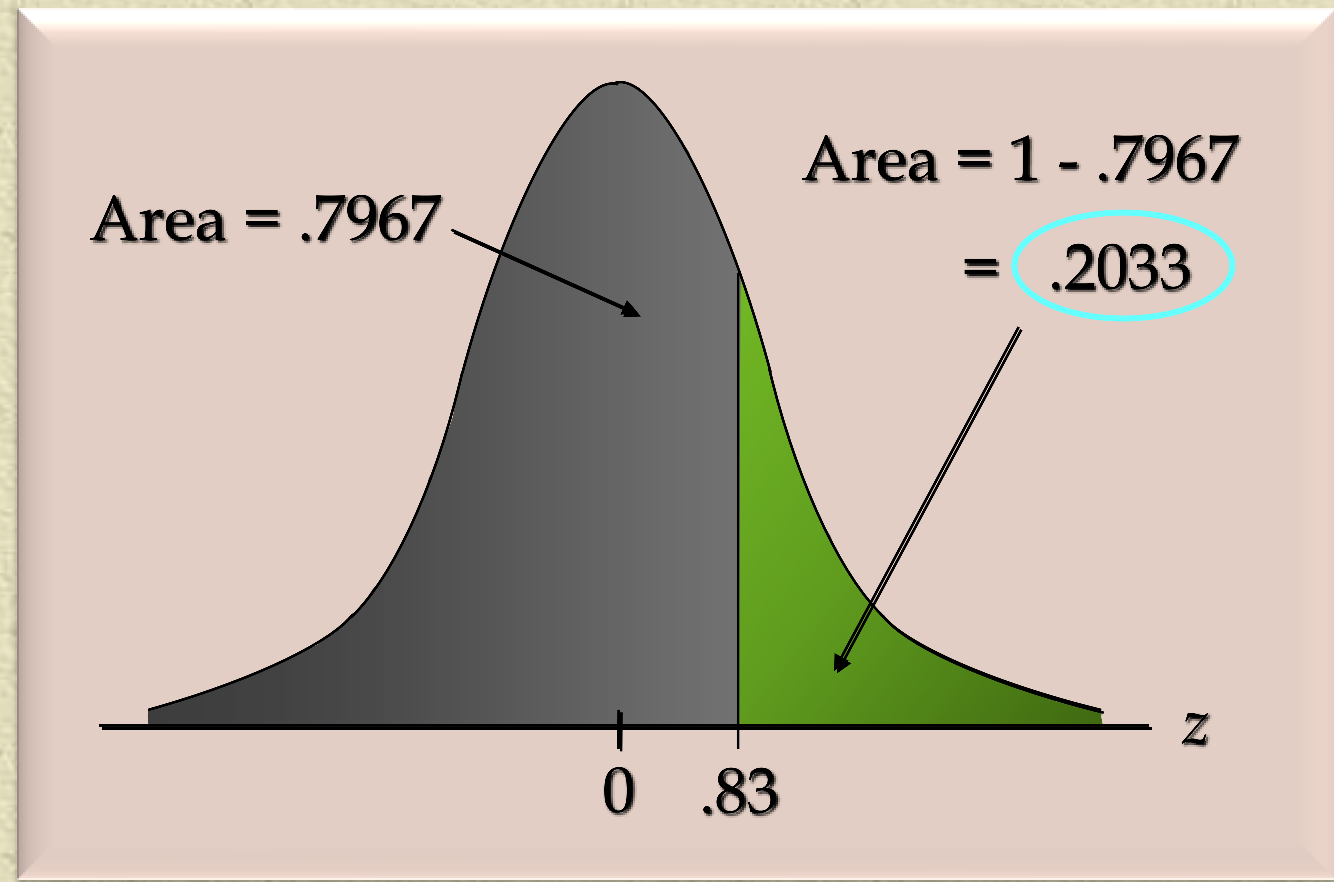
$$P(z > .83) = 1 - P(z \leq .83)$$
$$= 1 - .7967$$
$$= .2033$$

Probability of a stockout

$P(x > 20)$

Standard Normal Probability Distribution

- Solving for the Stockout Probability



Standard Normal Probability Distribution

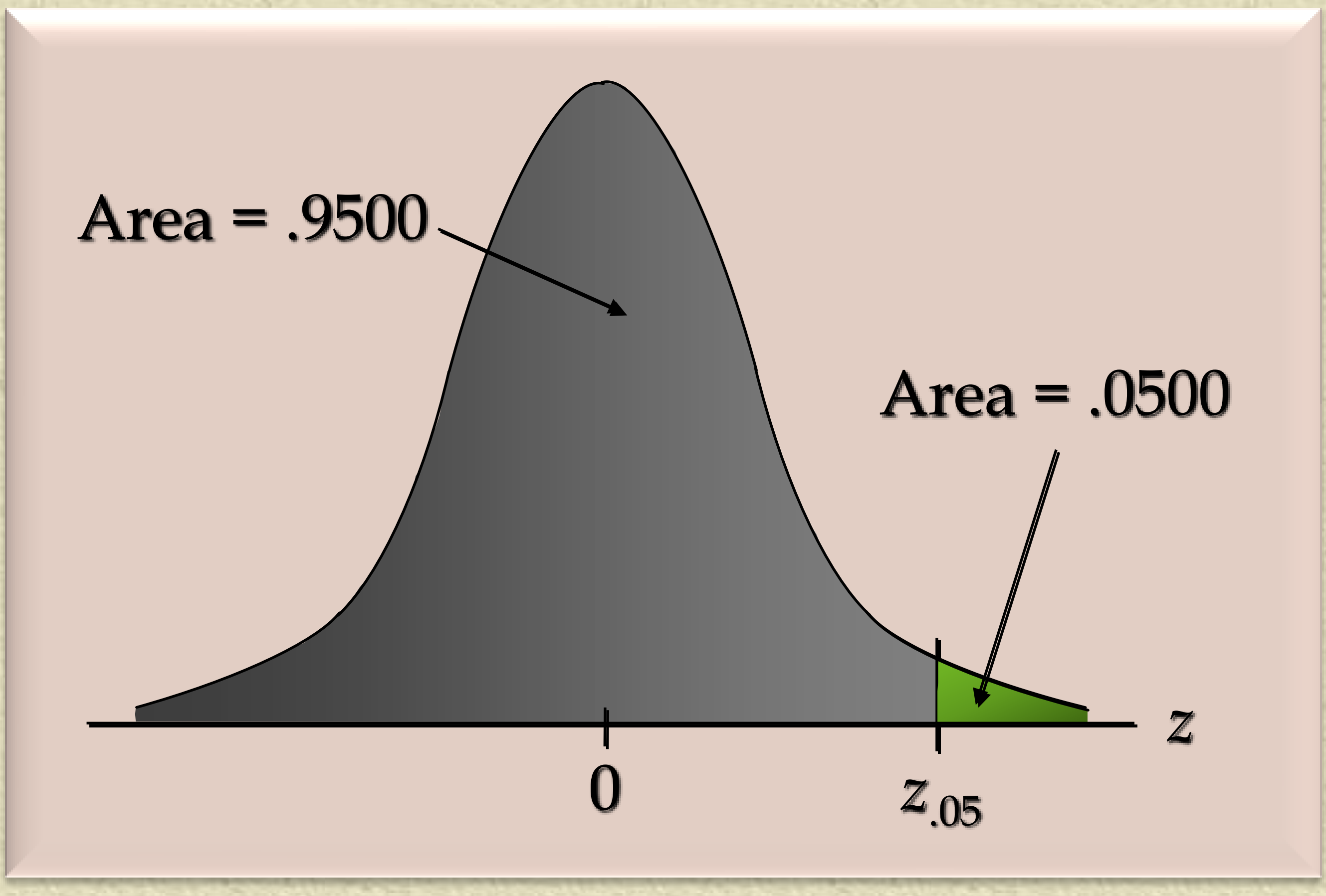
- Standard Normal Probability Distribution

  If the manager of Pep Zone wants the probability of a stockout during replenishment lead-time to be no more than .05, what should the reorder point be?

  (Hint: Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding *z* value.)

Standard Normal Probability Distribution

- Solving for the Reorder Point



Standard Normal Probability Distribution

- Solving for the Reorder Point

  - Step 1: Find the z-value that cuts off an area of .05 in the right tail of the standard normal distribution.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	[illegible]	[illegible]	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	[illegible]	[illegible]	[illegible]	[illegible]
1.9	.9713	.9719	.9726	.9732	.9738	.9744	[illegible]	[illegible]	[illegible]	[illegible]
.	.	.	.	.	.	.				

We look up the complement of the tail area (1 - .05 = .95)

Standard Normal Probability Distribution

- Solving for the Reorder Point

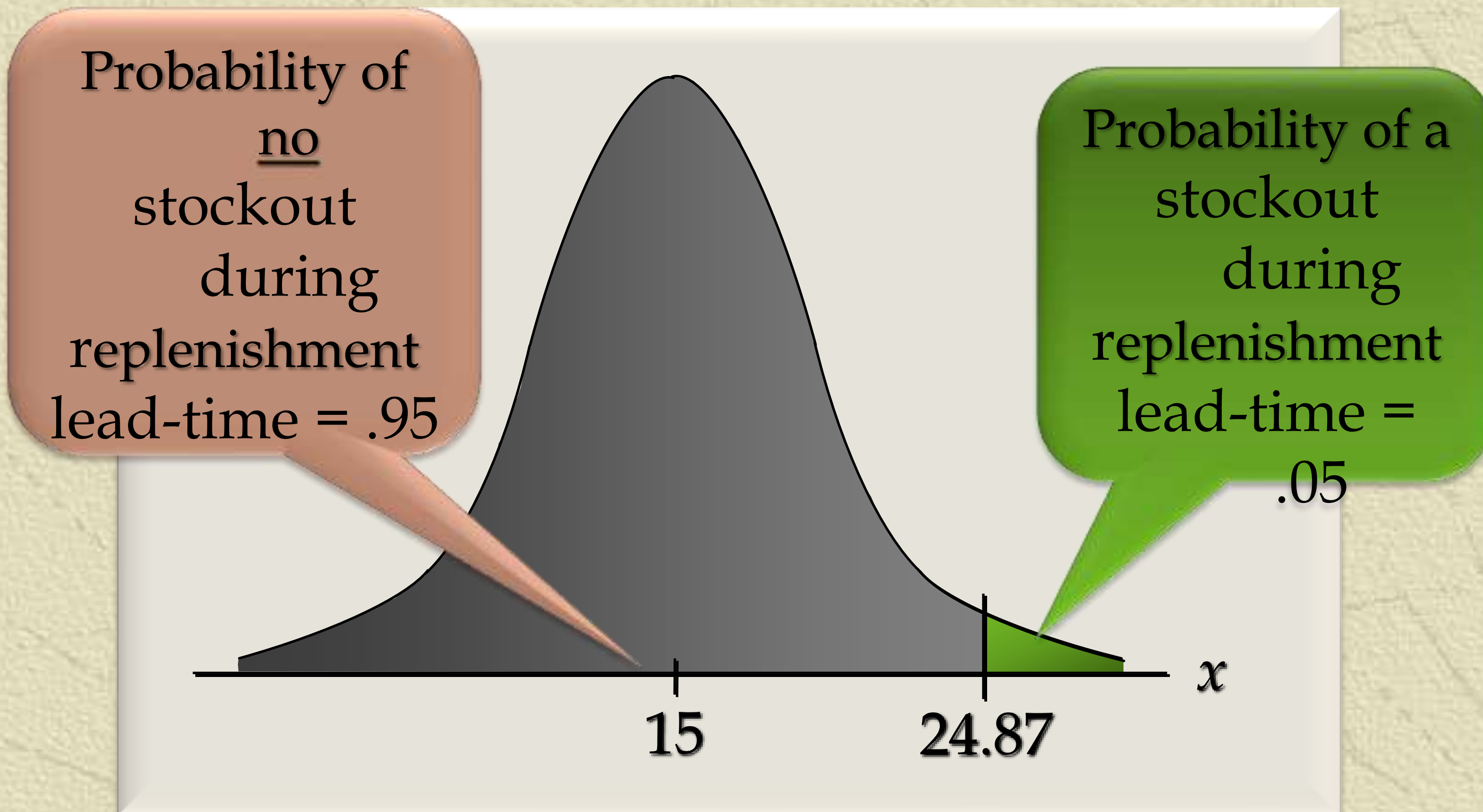
Step 2: Convert $z_{.05}$ to the corresponding value of x.

$$
\begin{aligned}
x &= \mu + z_{.05}\sigma \\
&= 15 + 1.645(6) \\
&= 24.87 \text{ or } 25
\end{aligned}
$$

A reorder point of 25 gallons will place the probability of a stockout during leadtime at (slightly less than) .05.

Normal Probability Distribution

- **Solving for the Reorder Point**



Standard Normal Probability Distribution

- Solving for the Reorder Point
- By raising the reorder point from 20 gallons to 25 gallons on hand, the probability of a stockout decreases from about .20 to .05.

  This is a significant decrease in the chance that Pep Zone will be out of stock and unable to meet a customer's desire to make a purchase.

Case-Let

- We have a training program designed to upgrade the supervisory skills of production line supervisors. Because the program is self administered, supervisors requires different no. of hours to complete the program.A study of past participants indicates the mean length of time spent on the program is 500 hrs and that this normally distributed random variable has a standard deviation of 100 hours.

- What is the probability that a participant selected at random will require more than 500 hrs to complete the program?
- What is the prob. That a candidate selected at random will take between 500 and 650 hours to complete the training program?
- What is the prob. That a candidate will take more than 700 hours to complete the program?
- Prob. In between 550 and 650 hrs.
- Fewer than 580 hrs.
- In between 420 and 680 hrs.

Q. A grinding machine is so set that its production of shafts has an average diameter of 10.10 cms and standard deviation of 0.20 cm. The product specifications call for shaft diameters between 10.05 cm and 10.20 cm. What proportion of output meets the specifications presuming normal distribution?

SOL: 0.2902

Q. Assume mean height of soldiers to be 68.22 inches with variance of 10.8 inches. How many soldiers in a regiment of 1,000 are Expected to be over six feet tall.

SOL: 125

Q.In a test given to 1,000 students, the average score was 42 and s.d = 24. Find

a) number of students exceeding a score of 50.

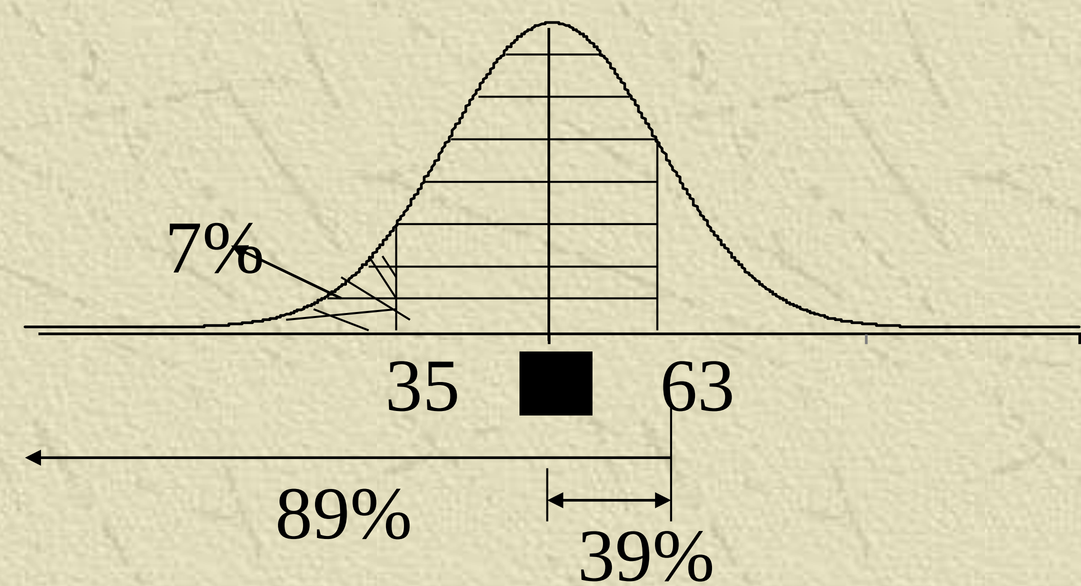
b) number of students between 30 and 54

SOL: A) 1000x .3696 B) 383

Q. Sacks of grain have an average of 120 kgs.It is found that 10% of the bags are over 125 kgs.Assuming the distribution to be normal,find the standard deviation.

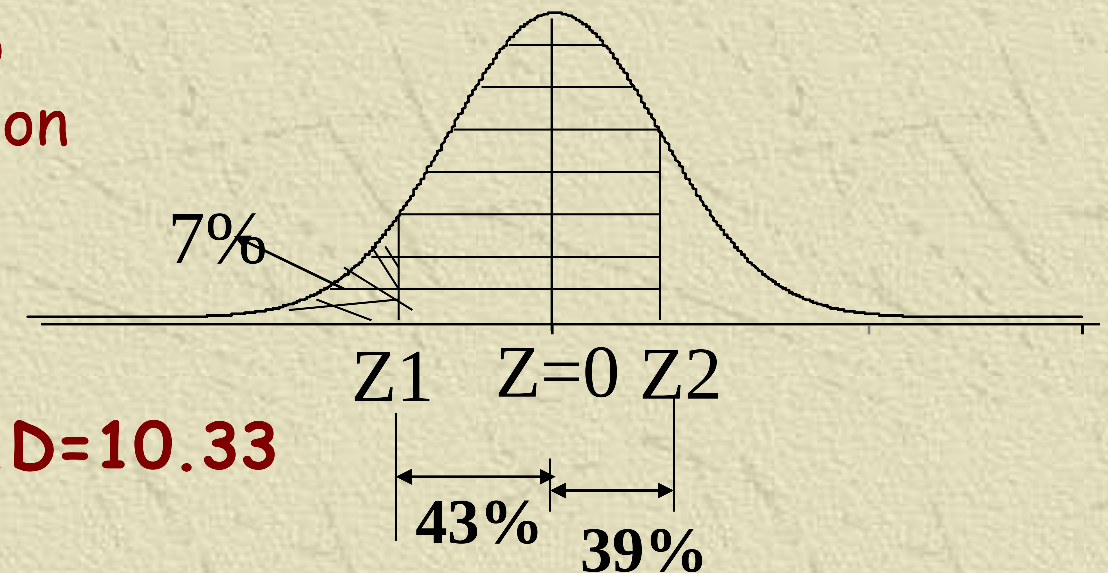
Q. Assume in a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What is the mean and s.D of the Distribution ?

SOL:



Z corresponding to X=35 and area .43 on Left is –1.48

Z corresponding to X=63 and area .39 on Right is 1.23



MEAN=50.3, S.D=10.33

Q. Of a large group of men 5% are under 60 inches in height and 40% are between 60 & 65 inches. Assuming a normal distribution, find mean height and s.d.

ANS: MEAN=65.429;S.D= 3.3

Normal Approximation of Binomial Probabilities

- When the number of trials, *n*, becomes large, evaluating the binomial probability function by hand or with a calculator is difficult.

- The normal probability distribution provides an easy-to-use approximation of binomial probabilities where $np \geq 5$ and $n(1 - p) \geq 5$.

- In the definition of the normal curve, set

$$\mu = np \quad \text{and} \quad \boxed{\sigma = \sqrt{np(1-p)}}$$

Normal Approximation of Binomial Probabilities

- Add and subtract a continuity correction factor because a continuous distribution is being used to approximate a discrete distribution.

- For example, $P(x = 12)$ for the discrete binomial probability distribution is approximated by $P(11.5 \leq x \leq 12.5)$ for the continuous normal distribution.

Normal Approximation of Binomial Probabilities

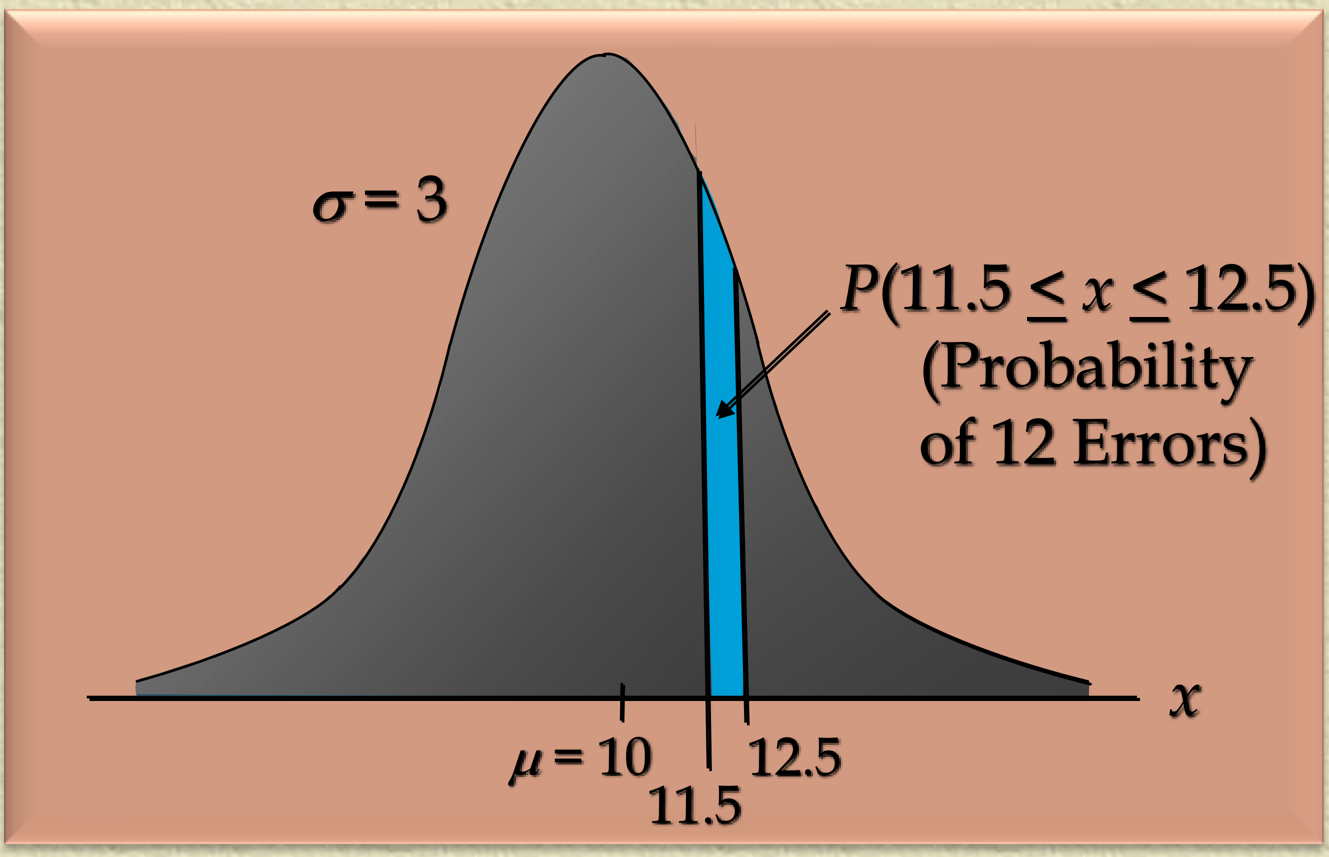
- Example
- Suppose that a company has a history of making errors in 10% of its invoices. A sample of 100 invoices has been taken, and we want to compute the probability that 12 invoices contain errors.
- In this case, we want to find the binomial probability of 12 successes in 100 trials. So, we set:

$$\mu = np = 100(.1) = 10$$

$$\sigma = \sqrt{np(1-p)} = [100(.1)(.9)]^{½} = 3$$

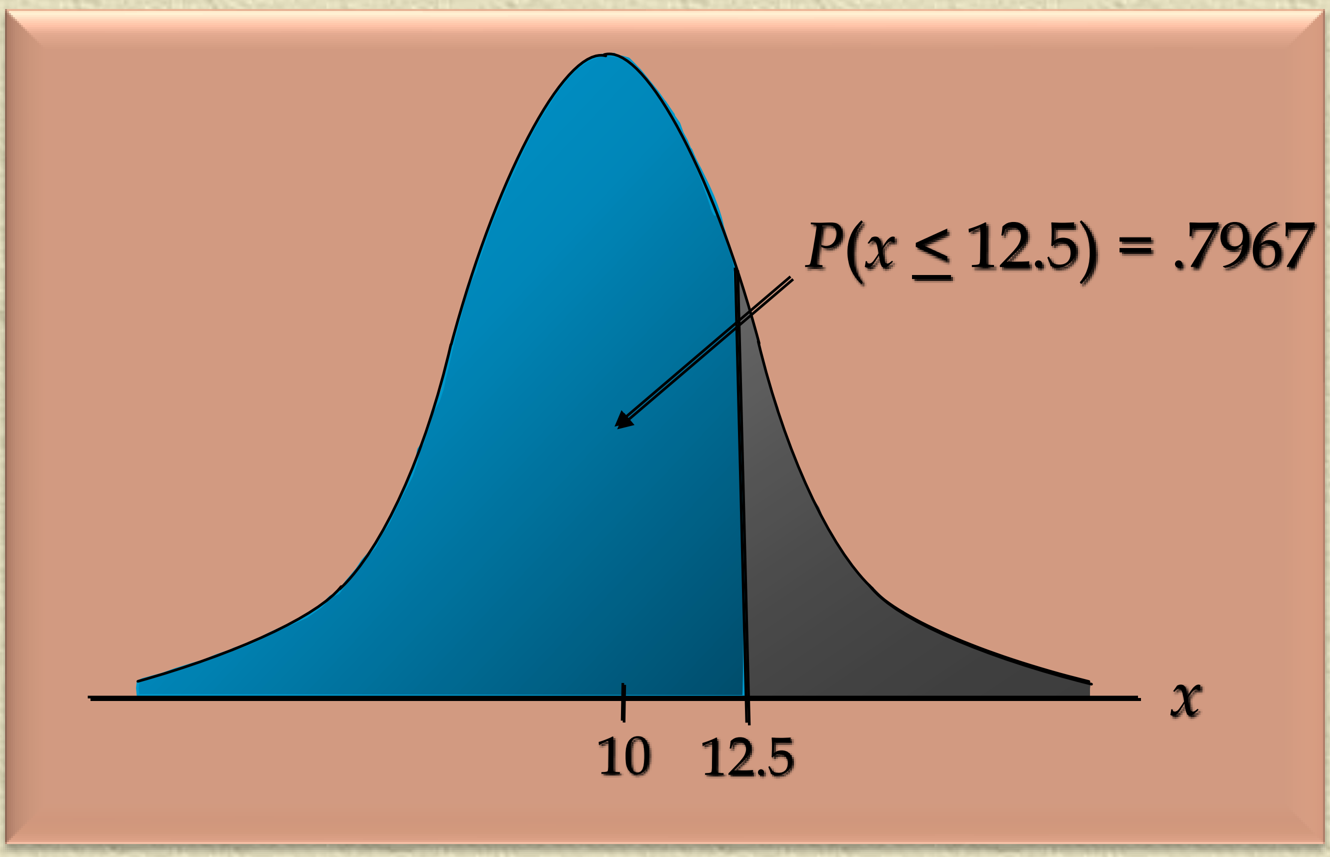
Normal Approximation of Binomial Probabilities

- Normal Approximation to a Binomial Probability Distribution with n = 100 and p = .1



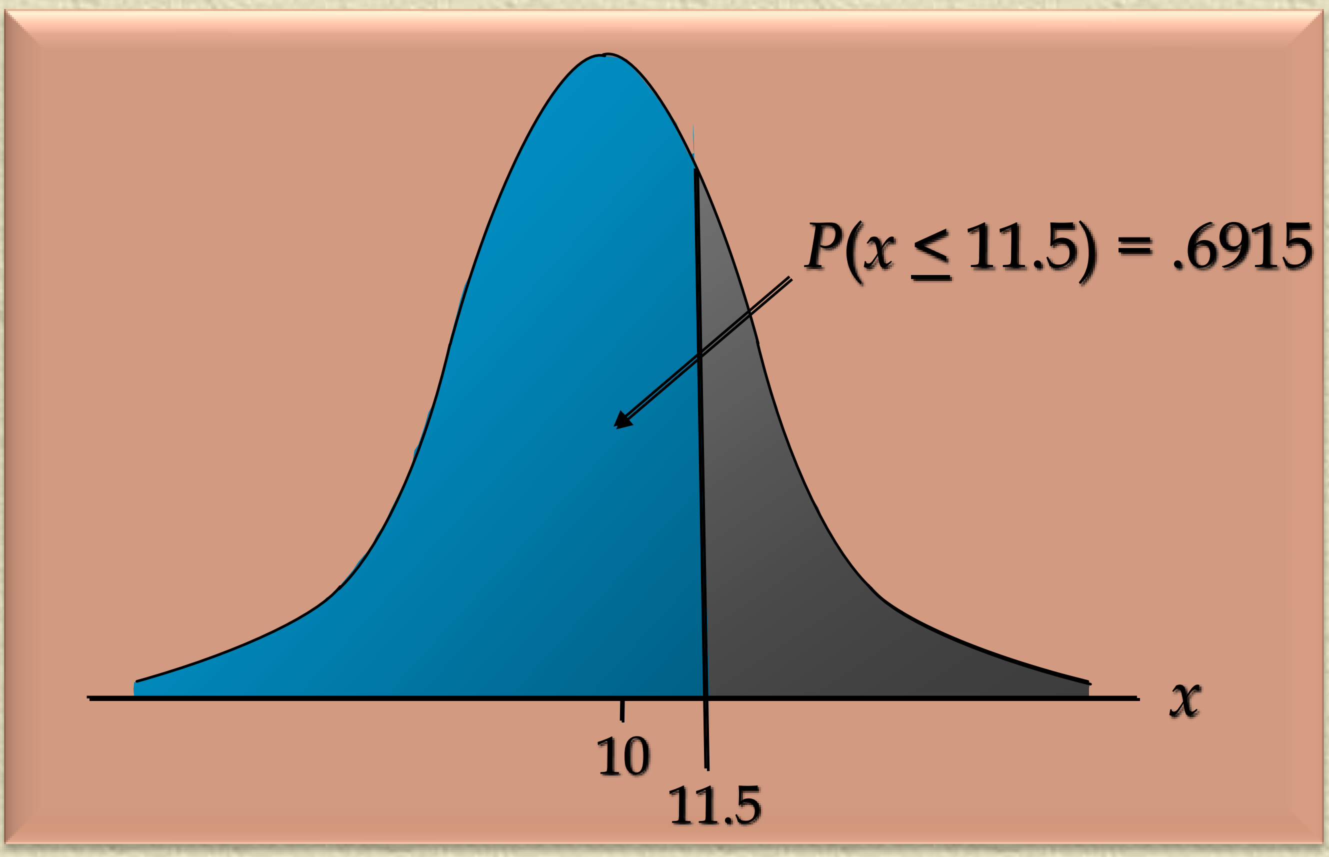
Normal Approximation of Binomial Probabilities

- Normal Approximation to a Binomial Probability Distribution with n = 100 and p = .1



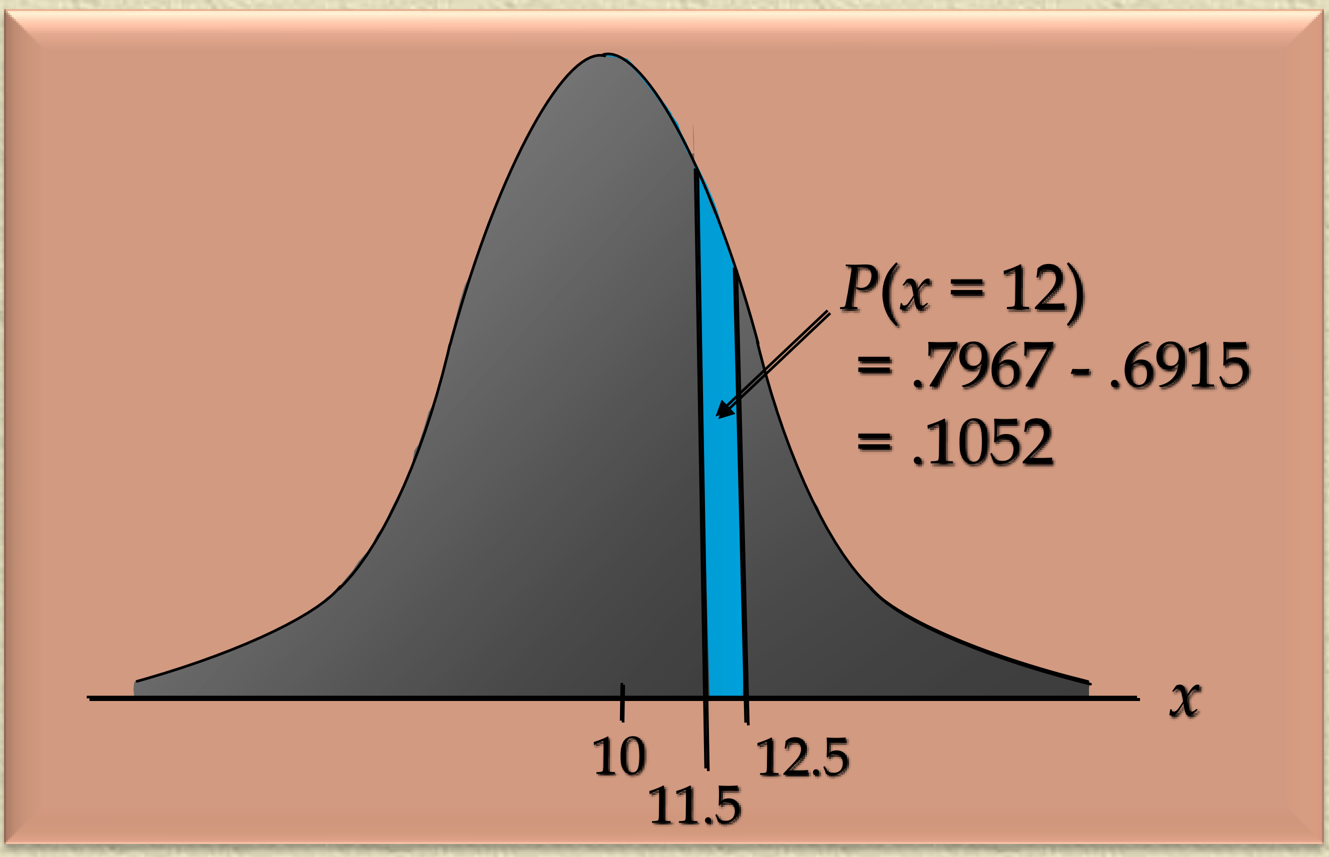
Normal Approximation of Binomial Probabilities

- Normal Approximation to a Binomial Probability Distribution with n = 100 and p = .1



Normal Approximation of Binomial Probabilities

- The Normal Approximation to the Probability of 12 Successes in 100 Trials is .1052



NORMAL APPROXIMATION

Q. Use Normal Approximation to the binomial distribution to find the probability of winning at most 70 of 100 matches by a team, when the probability of winning each match is .75

HINT: p= .75, n=100, MEAN=np=.75*100=75, S.D= SQRT(npq) = 4.33

Convert Discrete To Continuous- Apply Continuity Correction

Prob Of Winning Not More Than 70 Matches

$\cong P(X \leq 70.5) = P(Z \leq -1.04) = .15$

Q. Find the probability of getting 5 heads and 10 tails in 15 tosses of a fair coin using Normal distribution. Also compare your Answer with binomial distribution .

Sol: p be the prob of getting head in single toss =1/2

n=15 ; np=7.5 ; s.d = sqrt(npq)=1.94

Normal Approx: $P(X =5) \cong P(4.5 \leq X \leq 5.5)$

$= P(-1.55 \leq Z \leq -1.03) = .0909$

Binomial Distribution:

P(x=5)= $^{n}c_5 p^5 q^{n-5}$ = .09164 (approx. same)

Exponential Probability Distribution

- The exponential probability distribution is useful in describing the time it takes to complete a task.
- The exponential random variables can be used to describe:
  - Time between vehicle arrivals at a toll booth
  - Time required to complete a questionnaire
  - Distance between major defects in a highway
- In waiting line applications, the exponential distribution is often used for service times.

Exponential Probability Distribution

- A property of the exponential distribution is that the mean and standard deviation are equal.

- The exponential distribution is skewed to the right.

Exponential Probability Distribution

- Density Function

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \quad \text{for } x \geq 0$$

where: μ = expected or mean

e = 2.71828

Exponential Probability Distribution

- Cumulative Probabilities

$$P(x \leq x_0) = 1 - e^{-x_0/\mu}$$

where:

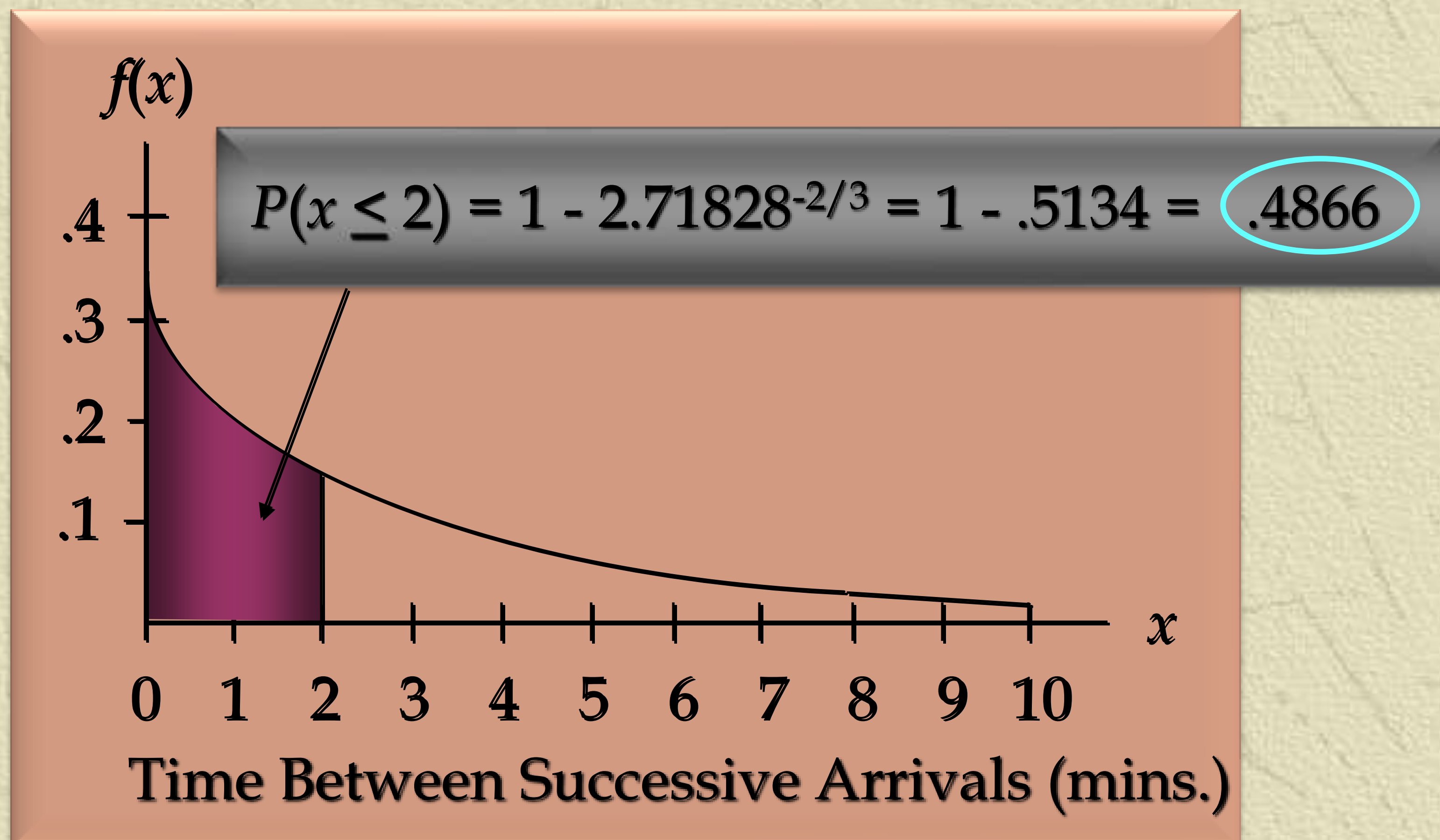
x_0 = some specific value of x

Exponential Probability Distribution

- Example: Al's Full-Service Pump
- The time between arrivals of cars at Al's full-service gas pump follows an exponential probability distribution with a mean time between arrivals of 3 minutes. Al would like to know the probability that the time between two successive arrivals will be 2 minutes or less.

Exponential Probability Distribution

- Example: Al's Full-Service Pump



Relationship between the Poisson and Exponential Distributions

- The Poisson distribution provides an appropriate description of the number of occurrences per interval
- The exponential distribution provides an appropriate description of the length of the interval between occurrences

Example

- Suppose the umber of cars that arrive at a car wash during one hour is described by Poisson probability distribution with a mean of 10 cars per hr. The Poisson probabixlity function that gives the probability of x arrivals per hour is

$$f(x)= 10^{x} e^{-10}/ x!$$

- As the average number of arrivals is 10 cars hour, the average time between cars arriving is
- 1hr/10 cars = 0.1hr/car
- It implies the time between the arrivals has a mean =0.1 hr per car, so the exponential distribution is
- $f(x) = 10e^{-10x}$

32. Consider the following exponential probability density function.

$$f(x) = \frac{1}{8} e^{-x/8} \qquad \text{for } x \geq 0$$

a. Find $P(x \leq 6)$.
b. Find $P(x \leq 4)$.
c. Find $P(x \geq 6)$.
d. Find $P(4 \leq x \leq 6)$.

33. Consider the following exponential probability density function.

$$f(x) = \frac{1}{3} e^{-x/3} \qquad \text{for } x \geq 0$$

a. Write the formula for $P(x \leq x_0)$.
b. Find $P(x \leq 2)$.
c. Find $P(x \geq 3)$.
d. Find $P(x \leq 5)$.
e. Find $P(2 \leq x \leq 5)$.

32. a. $P(x \le 6) = 1 - e^{-6/8} = 1 - .4724 = .5276$

b. $P(x \le 4) = 1 - e^{-4/8} = 1 - .6065 = .3935$

c. $P(x \ge 6) = 1 - P(x \le 6) = 1 - .5276 = .4724$

d. $P(4 \le x \le 6) = P(x \le 6) - P(x \le 4) = .5276 - .3935 = .1341$

33. a. $P(x \le x_0) = 1 - e^{-x_0/3}$

b. $P(x \le 2) = 1 - e^{-2/3} = 1 - .5134 = .4866$

d. $P(x \le 5) = 1 - e^{-5/3} = 1 - .1889 = .8111$

e. $P(2 \le x \le 5) = P(x \le 5) - P(x \le 2) = .8111 - .4866 = .3245$

37. Collina's Italian Café in Houston, Texas, advertises that carryout orders take about 25 minutes (Collina's website, February 27, 2008). Assume that the time required for a carryout order to be ready for customer pickup has an exponential distribution with a mean of 25 minutes.

a. What is the probability than a carryout order will be ready within 20 minutes?
b. If a customer arrives 30 minutes after placing an order, what is the probability that the order will not be ready?
c. A particular customer lives 15 minutes from Collina's Italian Café. If the customer places a telephone order at 5:20 P.M., what is the probability that the customer can drive to the café, pick up the order, and return home by 6:00 P.M.?

37. a. $f(x) = \frac{1}{\mu} e^{-x/\mu} = \frac{1}{25} e^{-x/25}$ for $x \geq 0$

$P(x \leq x_0) = 1 - e^{-x_0/\mu}$

$P(x \leq 20) = 1 - e^{-20/25} = 1 - e^{-.80} = 1 - .4493 = .5507$

b. $P(x > 30) = 1 - P(x \leq 30) = 1 - (1 - e^{-30/25}) = e^{-1.2} = .3012$

c. For the customer to make the 15-minute return trip home by 6:00 p.m., the order must be ready by 5:45 p.m. Since the order was placed at 5:20 p.m., the order must to be ready within 25 minutes.

$P(x \leq 25) = 1 - e^{-25/25} = 1 - e^{-1} = 1 - .3679 = .6321$

This may seem surprising high since the mean time is 25 minutes. But, for the exponential distribution, the probability x being greater than the mean is significantly less than the probability of x being less than the mean. This is because the exponential distribution is skewed to the right.